Material Design: Fiber Orientation for a Composite Exposed to a Fatigue Load

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Introduction

Composite materials, such as carbon fiber or fiberglass reinforced polymers, are not isotropic. Their strength and stiffness depend heavily on the direction of the applied load relative to the fiber orientation. This anisotropy is a powerful design tool. For components subjected to **fatigue loads** (cyclic stresses), choosing the correct fiber orientation can dramatically increase the lifespan by reducing the critical stresses that cause damage initiation in the matrix or at the fiber-matrix interface.

This set of notes outlines a systematic method, implemented in a MATLAB program, to determine relative optimum fiber orientation for common engineering components under fatigue loading.

Components and Loads Considered

The methodology is applicable to the following scenarios:

- **Component Geometries:**
  - **Plate:** A simple rectangular beam or plate.
  - **Solid/Hollow Shaft:** A structural element primarily transmitting torque.
  - **Pressure Vessel/Tube:** A cylindrical vessel subjected to internal pressure.
- **Fatigue Load Types:**
  - **Bending Moment (M):** Cyclic bending.
  - **Torsional Torque (T):** Cyclic twisting.
  - **Axial Force (F):** Cyclic tension or compression.
  - **Internal Pressure (p):** Cyclic pressure.

Key Concepts in Fatigue Analysis

- **Cyclic Load Characterization:** Any cyclic load can be decomposed into two parts:
  - **Mean Load (Lmean*Lmean*):** The steady component. Lmean=(Lmax+Lmin)/2
  - **Amplitude Load (Lamp*Lamp*):** The alternating component. Lamp=(Lmax−Lmin)/2
- **Stress Ratio (R):** Defines the nature of the cycle. R=Lmin/Lmax
  - R=−1: Fully reversed (damaging for isotropic materials).
  - R=0: Pulsating tension.
  - The code calculates mean and amplitude from either (Lmax,Lmin) or (Lmean,R).

Global Stress Calculations

The first step is to calculate the stress state at critical point on the component (e.g., the outer surface). The code calculates stresses in a global Cartesian coordinate system (X, Y, Z).

- **Bending Stress (σ_{xb}):** $\sigma_{xb} = M\,y / I$
  - M = Bending Moment, *y* = distance from neutral axis, *I* = Area Moment of Inertia.
- **Axial Stress (σ_{xa}):** $\sigma_{xa} = F / A$
  - F*F* = Axial Force, A*A* = Cross-sectional Area.
- **Torsional Shear Stress (τ_{xy}):** $\tau_{xy} = T\,r / J$
  - T = Torque, r = radius to outer surface, *J* = Polar Moment of Inertia.
- **Pressure Vessel Stresses (Thin-Walled):**
  - **Hoop Stress (σ_y):** $\sigma_y = pD / 2t$
  - **Longitudinal Stress (σ_x):** $\sigma_x = p\,D / 4t$
  - *p* = Pressure, *D* = Inner Diameter, *t* = Wall Thickness.

The total global stress components used for transformation are:

$\sigma_{x,max} = \sigma_{x,mean} + \sigma_{x,amp}$, $\sigma_{y,max} = \sigma_{y,mean} + \sigma_{y,amp}$, $\tau_{xy,max} = \tau_{xy,mean} + \tau_{xy,amp}$

Stress Transformation to Ply Coordinates

For relative optimization, process is transforming the global stresses ($\sigma_x,\sigma_y,\tau_{xy}$) into the local coordinate system (1, 2, 6) of a unidirectional ply, where:

- **1-direction:** Parallel to the fibers.
- **2-direction:** Perpendicular to the fibers.
- **6-direction:** In-plane shear stress.

The transformation depends on the ply orientation angle, θ, measured from the global x-axis to the local 1-direction. The transformation equations are:

$\sigma_1=\sigma_x\cos2\theta+\sigma_y\sin2\theta+2\tau_{xy}\sin\theta\cos\theta$

$\sigma_2=\sigma_x\sin2\theta+\sigma_y\cos2\theta-2\tau_{xy}\sin\theta\cos\theta$

$\sigma_6=(-\sigma_x+\sigma_y)\ \sin\theta\cos\theta+\tau_{xy}(\cos2\theta-\sin2\theta)$

These equations are vectorized in the MATLAB code to calculate $\sigma_1,\sigma_2,\sigma_6$ for all angles from 0° to 90°.

Overview of the MATLAB Program

The shared code follows a logical sequence of user input, calculation, and visualization.

Workflow Steps:

1. **Initialization:** Sets all stress and load variables to zero to avoid errors from undefined variables.
2. **Load Assignment (Interactive):**
   - Asks the user which loads are present.
   - For each "yes," it prompts for maximum/minimum values or mean value and R-ratio.
   - Calculates the corresponding mean and amplitude components.
3. **Geometry Assignment (Interactive):**
   - Asks the user to select a shape (Plate, Vessel, Shaft).
   - Prompts for the necessary dimensions (e.g., width, thickness, diameters).
   - **Calculates Key Geometric Properties:**
     - Cross-sectional Area (A).
     - Area Moment of Inertia (I).
     - Polar Moment of Inertia (J).
     - Distance to outer fiber (y) and radius (r).
4. **Global Stress Calculation:**
   - Uses the formulas from Section 4 to compute σx, σy, and τxy using the *maximum* values of the loads. This gives the peak stress state for the fatigue cycle.
5. **Local Stress Calculation & Plotting:**
   - Defines an array of angles: Theta = 0:1:90.
   - Uses the transformation equations to calculate Sigma1Max, Sigma2Max, and Sigma6Max for every angle in Theta.
   - Plots all three local stresses on the same graph against the orientation angle.

Interpreting the Output

The final plot is the primary result. The x-axis is the fiber orientation angle θ (0° to 90°). The y-axis is the stress magnitude. By inspecting the lines for $\sigma 1$, $\sigma 2$, and σ6$\sigma 6$, an engineer can:

- Identify which stress component is highest (often the critical one for fatigue failure).
- Find the angle θ where this critical stress is at a **minimum**.
- This angle represents the **theoretically relative optimum fiber orientation** for resisting the specific combination of fatigue loads applied to the component.

Solved Example (From Code Output)

The interaction recorded at the end of the code is a solved task.

- **Task Definition:**
  - **Component:** A hollow shaft (Inner Diameter = 180 mm, Outer Diameter = 200 mm).
  - **Loads:** Fully reversed bending (Mmax=105 N·mm, Mmin=−105 N·mm) and fully reversed torsion (Tmax=105 N·mm, Tmin=−105 N·mm).
  - **Implied R-ratio:** R=−1 for both moment and torque.
- **Interpretation of Results:**
  - The code calculates the geometric properties (I, J, A, y, r) and the global stresses σx (from bending) and τxy (from torsion). σy is zero as there is no pressure.
  - It then generates a plot of σ1,σ2,σ6 vs. fiber orientation θ.
  - **To find relative optimum orientation:** The engineer would look at this plot. For instance, if the composite material is particularly weak in shear, minimizing σ6$\sigma 6$ would be the goal. The plot shows at which angle $\sigma 6$ is lowest. Conversely, if failure is driven by transverse tension, minimizing $\sigma 2$ would be the goal.

Conclusion

This methodology offers a powerful and practical approach for the preliminary design of composite components under fatigue. The MATLAB code automates the tedious calculations of stress transformation, allowing the designer to focus on interpreting the results.

Key Takeaway: fiber orientation is a compromise that depends on the specific combination of loads and the failure criteria of the composite material. This tool enables designers to make informed, relative optimal decisions to enhance fatigue life.

Material Design

Fiber Orientation for a Composite Exposed to a Fatigue Load

Introduction:

The program calculates relative optimum fiber orientation for the following samples geometry:

a) Plate
b) Solid shaft
c) Hollow shaft or pressure vessel or tube

Subjected to the following fatigue loads:

a) Bending
b) Torsion
c) Axial
d) Pressure

The case includes:

a) The matlab code
b) A solved problem in the section by the code

script

```
clear all
clc
%initialization
MeanMoment=0;
AmplitudeMoment=0;
MeanTorque=0;
AmplitudeTorque=0;
AmplitudePresuure=0;
MeanPresuure=0;
MeanForce=0;
AmplitudeForce=0;
VesselDiameter=0;
VesselThickness=1;
%loads assignment
ask1=input('Do you have a bending moment? please enter 1 for yes and 2 for no :\n');
if ask1==1
MaxMoment=input('enter the max moment in newton millimeter:\n');
MinMoment=input('enter the min moment in newton millimeter:\n');
Rm=MinMoment/MaxMoment;
MeanMoment=(MaxMoment+MinMoment)/2;
AmplitudeMoment=(MaxMoment-MinMoment)/2;
```

```
end
ask2=input('Do you have a torsional torque ? please enter 1 for yes and 2 for no :\n');
if ask2==1
MaxTorque=input('enter the max torque in newton millimeter :\n');
MinTorque=input('enter the min torque in newton millimeter :\n');
Rt=MinTorque/MaxTorque;
MeanTorque=(MaxTorque+MinTorque)/2;
AmplitudeTorque=(MaxTorque-MinTorque)/2;
end
ask3=input('Do you have an axial force ? please enter 1 for yes and 2 for no :\n');
if ask3==1
MeanForce=input('enter the mean force in newton millimeter :\n');
Rf=input('enter the stress ratio :\n');
AmplitudeForce=MeanForce*(1-Rf)/(1+Rf);
MaxForce=AmplitudeForce+MeanForce;
MinForce=Rf*MaxForce;
end
ask4=input('Do you have a pressure ? please enter 1 for yes and 2 for no :\n');
if ask4==1
AmplitudePresuure=input('enter the amplitude pressure in mega pascal :\n');
Rf=input('enter the stress ratio :\n');
MeanPresuure=AmplitudePresuure*(1+Rf)/(1-Rf);
MaxPresuure=AmplitudePresuure+MeanPresuure;
MinPresuure=Rf*MaxPresuure;
end
%sample assignment
ask1=input('what is the design shape ? please enter 1 for plate , 2 for pressure vessel and 3 for
shaft :\n');
if ask1==1
PlateLength=input('enter the plate length in millimeters :\n');
PlateWidth=input('enter the plate width in millimeters :\n');
PlateThickness=input('enter the plate thickness in millimeters :\n');
I1=(PlateThickness*PlateWidth^3)/12;
I2=(PlateWidth*PlateThickness^3)/12;
I=I2;
J=I1+I2;
A=PlateWidth*PlateThickness;
y=PlateThickness;
r=PlateThickness;
elseif ask1==2
VesselDiameter=input('enter the vessel inner diameter in millimeters :\n');
VesselThickness=input('enter the vessel thickness in millimeters :\n');
VesselLength=input('enter the vessel length in millimeters :\n');
I=pi/64*((VesselDiameter+2*VesselThickness)^4-VesselDiameter^4);
J=pi/32*((VesselDiameter+2*VesselThickness)^4-VesselDiameter^4);
r=VesselDiameter/2;
```

```
y=VesselDiameter/2;
A=pi/4*((VesselDiameter+2*VesselThickness)^2-VesselDiameter^2);
elseif ask1==3
ShaftInnerDiameter=input('enter the shaft inner diameter in millimeters :\n');
ShaftOuterDiameter=input('enter the shaft outer diameter in millimeters :\n');
ShaftLength=input('enter the shaft length in millimeters:\n');
I=pi/64*(ShaftOuterDiameter^4-ShaftInnerDiameter^4);
J=pi/32*(ShaftOuterDiameter^4-ShaftInnerDiameter^4);
y=ShaftOuterDiameter/2;
r=ShaftOuterDiameter/2;
A=pi/4*(ShaftOuterDiameter^2-ShaftInnerDiameter^2);
end
%global stresses calculations
SigmaMeanX=MeanMoment*y/I+MeanForce/A+MeanPresuure*VesselDiameter/4/VesselThick
ness;
SigmaAmplitudeX=AmplitudeMoment*y/I+AmplitudeForce/A+AmplitudePresuure*VesselDia
meter/4/VesselThickness;
SigmaMaxX=SigmaMeanX+SigmaAmplitudeX;
TawMean=MeanTorque*r/J;
TawAmplitude=AmplitudeTorque*r/J;
TawMax=TawMean+TawAmplitude;
SigmaMeanY=MeanPresuure*VesselDiameter/2/VesselThickness;
SigmaAmplitudeY=AmplitudePresuure*VesselDiameter/2/VesselThickness;
SigmaMaxY=SigmaMeanY+SigmaAmplitudeY;
%local streses plotting with respect to orientation angle
Theta=0:1:90;
Sigma1Max=(((cosd(Theta)).^2)*SigmaMaxX)+(((sind(Theta)).^2)*SigmaMaxY)+(sind(2*Thet
a)*TawMax);
Sigma2Max=((sind(Theta)).^2*SigmaMaxX)+((cosd(Theta)).^2*SigmaMaxY)+(-
sind(2*Theta)*TawMax);
Sigma6Max=(-
sind(2*Theta)/2*SigmaMaxX)+(sind(2*Theta)/2*SigmaMaxY)+(cosd(2*Theta)*TawMax);
plot(Theta,Sigma1Max,'b');
hold on
plot(Theta,Sigma2Max,'r');
hold on
plot(Theta,Sigma6Max,'y');
legend('Sigma1Max','Sigma2Max','Sigma6Max')
```

case

Do you have a bending moment? please enter 1 for yes and 2 for no:

1

enter the max moment in newton millimeter:

1e5

enter the min moment in newton millimeter:

-1e5

Do you have a torsional torque? please enter 1 for yes and 2 for no:

1

enter the max torque in newton millimeter:

1e5

enter the min torque in newton millimeter:

-1e5

Do you have an axial force? please enter 1 for yes and 2 for no:

2

Do you have a pressure? please enter 1 for yes and 2 for no:

2

what is the design shape? please enter 1 for plate, 2 for pressure vessel and 3 for shaft:

3

enter the shaft inner diameter in millimeters:

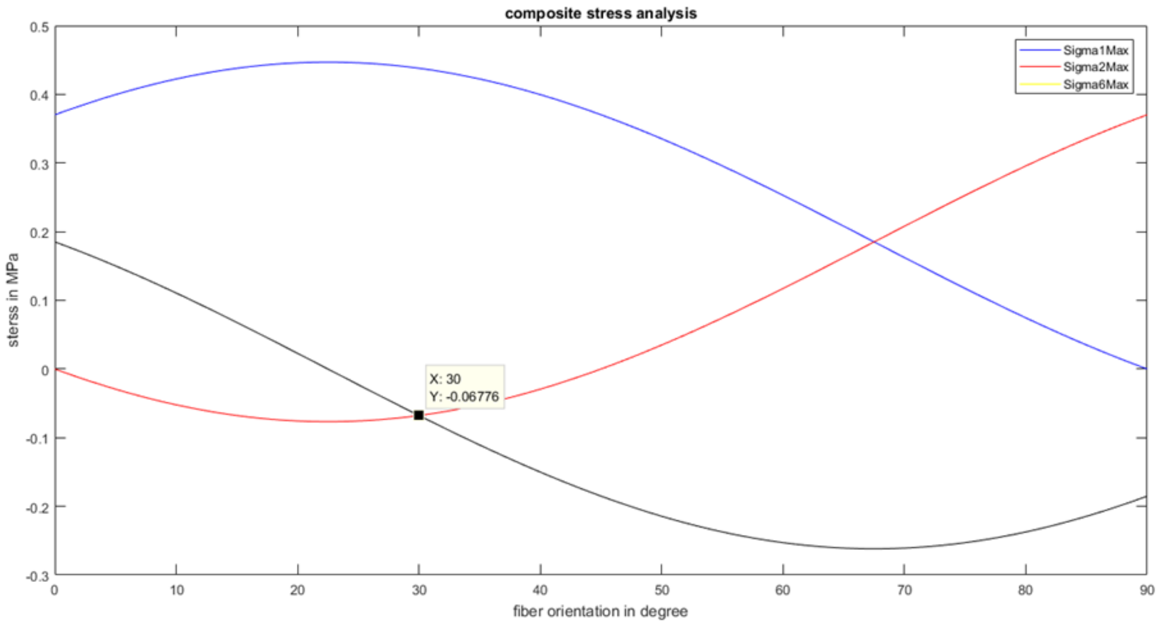
180

enter the shaft outer diameter in millimeters:

200

enter the shaft length in millimeters:

1


composite stress analysis
Sigma1Max
Sigma2Max
Sigma6Max
X: 30
Y: -0.06776
sterss in MPa
fiber orientation in degree
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
10
20
30
40
50
60
70
80
90

Quiz

Question 1
In the context of fatigue loading, what do the "mean" and "amplitude" components of a load represent?
a) The minimum and maximum load values.
b) The static (average) load and the dynamic (alternating) load component.
c) The force and the moment acting on a structure.
d) The axial and shear components of stress.

Question 2
For a hollow shaft subjected to torsion, which geometric property is most critical for calculating the shear stress?
a) The Area (A)
b) The Moment of Inertia (I)
c) The Polar Moment of Inertia (J)
d) The cross-sectional width

Question 3
In the stress transformation calculations within the code
(Sigma1Max, Sigma2Max, Sigma6Max), what does the variable Theta represent?
a) The angle of twist in the shaft.
b) The angle between the loading direction and the material's principal direction.
c) The phase angle between different loads.
d) The angle of internal friction of the composite.

Question 4
If a pressure vessel is selected as the geometry, the code calculates a stress component SigmaMeanY. What is the primary source of this stress?
a) Bending Moment
b) Axial Force
c) Hoop stress due to internal pressure
d) Torsional Shear

Question 5
The code calculates a stress ratio R (e.g., Rm, Rt) for some loads. If R = -1, what does this imply about the load cycle?
a) The load is fully reversed (equal magnitude in positive and negative directions).
b) The load is purely static (constant with time).
c) The load is a pulsating tension (varies between zero and a maximum value).
d) The mean stress is zero, but the amplitude is not necessarily maximum.

Question 6

What is the primary purpose of plotting Sigma1Max, Sigma2Max, and Sigma6Max against the angle Theta?

a) To determine the factor of safety of the structure.
b) To identify the fiber orientation that minimizes the most critical stress component for fatigue.
c) To calculate the natural frequency of the component.
d) To validate the global stress calculations against theoretical values.

Question 7

For a composite ply, which stress component is relatively most detrimental to fatigue life when it is high?

a) Sigma1 (stress parallel to fibers)
b) Sigma2 (stress transverse to fibers)
c) Sigma6 (in-plane shear stress)
d) They are all equally detrimental.

Question 8

In the shared example run of the code, the shaft is subjected to fully reversed bending and torsion. The resulting plot shows the stresses as a function of angle. At which angle would you expect the shear stress (Sigma6) to be zero for a case of pure bending?

a) 0°
b) 30°
c) 45°
d) 90°